

# Decomposition of Choices Correspondences

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## Abstract

A choice of actions by a company performing a project might be made by delegation by CEO to departments, and by VPs to sub-departments, etc. Thus the following natural question arises: Can we detect the structure of a corporation from the choice of actions on a project? Here we answer this question by carefully analyzing the internal structure of the choice correspondence that models the corporation's behavior.

A *choice (correspondence)* on a set  $X$  of alternatives is a function defined on each nonempty set  $A \subseteq X$  (a menu) with the property that  $\emptyset \neq c(A) \subseteq A$ . We call a choice  $c$  *preordered* whenever there exists a preorder  $\succsim$  (reflexive and transitive) on  $X$  such that the choice set  $c(A)$  of each menu  $A$  is equal to the set of non-dominated elements according to  $\succsim$ , that is,  $c(A) = \{x \in A : (\nexists y \in A)(y \succ x)\}$ . (In classical terms, this is the case of a quasi-transitively rationalizable choice, according the theory of revealed preferences pioneered by Samuelson (1938).)

Given choices  $(X, c_X)$  and  $\{(Y_x, c_{Y_x}) : x \in X\}$ , define a *corporate choice*  $c_Z$  on  $Z = \bigcup\{Y_x : x \in X\}$  by  $c_Z(A) = \bigcup\{c_{Y_x}(A \cap Y_x) : x \in c_X(\pi(A))\}$ , where  $\pi(A) = \{x \in X : A \cap Y_x \neq \emptyset\}$ . Then  $(Z, c_Z)$  is said to be *paged* into  $\{(Y_x, c_{Y_x}) : x \in (X, c)\}$ . Further, we say  $(Z, c_Z)$  has a *preordered book* if  $(X, c_X)$  is preordered, and has an *unpageable book* otherwise. Intuitively, a preordered book reveals a rational CEO, whereas an unpageable book reveals an irrational CEO.

We show that every choice on a finite set can be uniquely expressed as an iterated paging, in which points are paged into preordered or unpageable choices, but points of a preordered choice are never paged into a preordered choice. Therefore, any choice reveals a unique corporate hierarchy, in which the reasoning by each executive is either maximally rational or minimally irrational, but no rational executive reports to another rational executive.

The key difficulty is to characterize pagination. We say that a nonempty  $E \subseteq X$  is *pageable* if the following properties hold:

1.  $A \cap E \neq \emptyset \implies c(A) \setminus E = c(A \cup E) \setminus E$ ;
2.  $c(A) \cap E \neq \emptyset \implies c(A) \cap E = c(A \cap E)$ ;
3.  $A \cap E \neq \emptyset \implies [c(A \cup E) \cap E \neq \emptyset \iff c(A) \cap E \neq \emptyset]$ .

These properties characterize autonomous corporate departments. Along the way, we also show that the following facts hold:

- (i) a sub-department of a department acts like a department;
- (ii) the union of two intersecting corporate departments acts like a corporate department;
- (iii) when a company splits into two departments, they are either independent or one dominates the other;
- (iv) the union of two incomparable intersecting departments acts like a single department composed of three sub-departments all independent, or each dominating the next, or the reverse.

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